# Advanced Econometrics: Time Series Models

Syllabus: Version 2b (April 15, 2024)

Instructor: Mikkel Plagborg-Moller, mikkelpm@princeton.edu

Lectures: Mon/Wed 10.40am-12.10pm, JRR 201

Office hours: Wed 2.00–3.00pm, JRR 282

Website: https://princeton.instructure.com/courses/13700

**Description.** Concepts and methods of time series analysis and their applications to economics. Time series models to be studied include simultaneous stochastic equations, VAR, ARIMA, and state-space models. Methods to analyze trends, second-moment properties via the auto covariance function and the spectral density function, methods of estimation and hypothesis testing and of model selection will be presented. Kalman filter and applications as well as unit roots, cointegration, ARCH, and structural breaks models are also studied.

**Prerequisites.** ECO 517 and 518, or equivalent. Students from outside the Economics PhD program should contact the instructor to obtain permission to take the course.

**Material.** There is no required textbook for the course. Handouts will be made available on the website. Attached to this syllabus is a list of optional readings that are useful for a deeper understanding of the material in the first half of the course.

Some students might find it useful to have a textbook as an additional reference. Good reference books include:

Brockwell, P. J., and Davis, R. A. (1991). *Time Series: Theory and Methods*. 2nd edition, Springer. (Beautiful mathematical treatment of the classic theory of covariance stationary time series, but not aimed at economists.)

Davidson, J. (1994). Stochastic Limit Theory. Oxford University Press. (Thorough, technical treatment of stochastic limit theory for dependent data.)

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press. (Comprehensive reference for time series econometrics methods developed before the mid-1990s.)

Hayashi, F. (2000). *Econometrics*. Princeton University Press. (Accessible treatment of GMM and stochastic limit theory for time series data.)

Herbst, E. P., and Schorfheide, F., (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press. (Recent reference on Bayesian inference in structural time series models.)

Kilian, L., and Lütkepohl, H. (2017). Structural Vector Autoregressive Analysis. Cambridge University Press. (Recent reference on SVAR methods.)

Homework. Problem sets will be posted on the course website approximately every one or two weeks. Students are encouraged to collaborate on the problem sets, but answers and computer code must be typed up independently. The problem sets will be graded coarsely, i.e., a full score will be given as long as the work demonstrates dedication and thoughtfulness. I reserve the right to subtract points for sloppy exposition, including unreadable code. If you find a grading error, please resubmit your problem set along with a one-paragraph explanation; I reserve the right to re-grade the entire problem set.

**Project presentation.** In lieu of a midterm and final exam, students will present a project in the final week of the course. Students may choose to collaborate in groups of two. The project may be original work, an in-depth critical assessment of an existing paper from the literature, or somewhere in between. Students should receive prior approval from the instructor of their choice of project topic.

**Grading.** The final course grade will be a monotonic function of the average of (i) the average problem set score (50% weight) and (ii) the project presentation score (50% weight).

Code of conduct. All course activities, including class meetings and homework assignments, are subject to the university's academic code and code of conduct as detailed in the "Rights, Rules, Responsibilities" publication.

Accommodations for students with disabilities. Students must register with the Office of Disability Services (ODS) (ods@princeton.edu; 258-8840) for disability verification and determination of eligibility for reasonable academic accommodations. Requests for academic accommodations for this course need to be made at the beginning of the semester, or as soon as possible for newly approved students, and again at least two weeks in advance of any

needed accommodations in order to make arrangements to implement the accommodations. Please make an appointment to meet with the instructor in order to maintain confidentiality in addressing your needs. No accommodations will be given without authorization from ODS, or without advance notice.

# Important dates.

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Jan 29 (Mon): First class
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Mar 11 (Mon) and Mar 13 (Wed): No class due to Spring Recess

Apr 22 (Mon): Student project presentations

Apr 24 (Wed): Last class

Course outline. The following outline is preliminary and may change without warning.

- 1. Refresher: Stationary models.
  - i) Strict/covariance stationarity, projection.
  - ii) Lag operators, linear filters, VARMA.
  - iii) Wold decomposition.
  - iv) Likelihood factorization, maximum likelihood estimation.
  - v) Bayesian VARs.
  - vi) Model selection.
- 2. Spectral analysis.
  - i) Spectrum of linear filter.
  - ii) Estimation: sieve-VAR, periodogram smoothing.
- 3. Inference with weakly dependent data.
  - i) Central Limit Theorem, martingale difference sequences, mixing.
  - ii) Applications to GMM, moment matching.
  - iii) Bootstrap.
  - iv) Weak identification (time permitting).

- 4. Causal identification in macroeconomics.
  - i) SVMA, SVAR, relationship with structural models.
  - ii) Invertibility, recoverability.
  - iii) Identification through exclusion restrictions.
  - iv) Local Projection versus VAR estimation of impulse responses.
  - v) Identification using instruments/proxies.
  - vi) Partial identification through sign/magnitude restrictions.
  - vii) Identification through non-Gaussianity/heteroskedasticity.
  - viii) Nonlinear models.
- 5. State space models.
  - i) Linear-Gaussian models, Kalman filter and smoother.
  - ii) Basics of Markov Chain Monte Carlo.
  - iii) Estimation of linearized DSGE models.
  - iv) Regime switching.
- 6. Dynamic factor models.
  - i) State space approach.
  - ii) Principal components.
  - iii) FAVAR.
  - iv) Inference on number of factors.
- 7. Estimation of heterogeneous agent models.
  - i) Challenges when combining micro and macro data.
  - ii) Moment matching.
  - iii) Likelihood inference.
- 8. Functional Central Limit Theorem.
  - i) Testing for structural breaks.

- 9. Non-stationary models.
  - i) I(1) processes, Beveridge-Nelson decomposition, VARIMA.
  - ii) Spurious regression.
  - iii) Bayesian vs. frequentist perspective.
  - iv) Frequentist asymptotics for unit roots, local-to-unity.
  - v) Cointegration, VECM.
  - vi) Detrending.
  - vii) Long-run inference (time permitting).
- 10. Long-run variance estimation (time permitting).
  - i) VAR-HAC.
  - ii) Spectral estimators.
  - iii) Kernel estimators.
  - iv) HAR inference.
- 11. Impulse response inference (time permitting).
  - i) Simultaneous confidence bands.
  - ii) Persistent data and long horizons.
- 12. Advanced Bayesian computation (time permitting).
  - i) Hamiltonian Monte Carlo.
  - ii) Particle filter.
  - iii) Sequential Monte Carlo.

# Optional reading list

Introductory readings are listed first and marked with a star (\*). Other readings are included for your reference. Original contributions are not always cited when good survey references are available. The reading list is preliminary and may change without warning.

# 1 Stationary models

# Models, prediction, estimation

\* Hayashi: chapters 6.1–6.4.

\* Kilian and Lütkepohl: chapters 2.1–2.5.

Brockwell and Davis: chapters 1.1–1.5, 2.1–2.9, 3.1–3.5, 5.1–5.5, 5.7, 11.1–11.4.

Hamilton: chapters 2-4, 10-12.

Herbst and Schorfheide: chapters 3.1–3.2.

Kilian and Lütkepohl: chapters 2, 5.

Giannone, D., Lenza, M., and Primiceri, G. E. (2015). "Prior Selection for Vector Autoregressions." Review of Economics and Statistics 97(2), 436–451.

#### Model selection

\* Kilian and Lütkepohl: chapter 2.6.

Brockwell and Davis: chapter 9.

Claeskens, G., and Hjort, N. L. (2008). *Model Selection and Model Averaging*. Cambridge University Press. Chapters 1–4.

Geweke, J., and Meese, R. (1981). "Estimating regression models of finite but unknown order." *International Economic Review* 22(1), 55–70.

Hansen, B. E. (2005). "Challenges for Econometric Model Selection." *Econometric Theory* 21(1), 60–68.

Leeb, H., and Pötscher, B. M. (2005). "Model Selection and Inference: Facts and Fiction." *Econometric Theory* 21(01), 21–59. Sections 1–2.

## **Applications**

- \* Stock, J. H., and Watson, M. W. (2017). "Twenty Years of Time Series Econometrics in Ten Pictures." *Journal of Economic Perspectives* 31(2), 59–86.
  - Bernanke, B. S., and Kuttner, K. N. (2005). "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *Journal of Finance* 60(3), 1221–1257.
  - Sims, C. A. (1972). "Money, Income, and Causality." *American Economic Review* 62(4), 540–552.
  - Stock, J. H., and Watson, M. W. (2003). "Forecasting Output and Inflation: The Role of Asset Prices." *Journal of Economic Literature* 41(3), 788–829.

# 2 Spectral analysis

# Representation theory and inference

\* Hamilton: chapter 6.

Brockwell and Davis: chapters 4, 10.1–10.5, 11.6.

Berk, N. (1974). "Consistent Autoregressive Spectral Estimates." Annals of Statistics 2(3), 489–502.

Hannan, E. J. (1970). Multiple Time Series. John Wiley & Sons. Chapters III.2–3, III.5.

# Applications

- Dew-Becker, I., and Giglio, S. (2016). "Asset Pricing in the Frequency Domain: Theory and Empirics." Review of Financial Studies 29(8), 2029–2068.
- King, R. G., and Watson, M. W. (1996). "Money, Prices, Interest Rates and the Business Cycle." *Review of Economics and Statistics* 78(1), 35–53.
- Qu, Z., and Tkachenko, D. (2012). "Frequency Domain Analysis of Medium Scale DSGE Models with Application to Smets and Wouters (2007)." In Advances in Econometrics, Volume 28: DSGE Models in Macroeconomics Estimation, Evaluation and New Developments, edited by Balke, N., Canova, F., Milani, F., and Wynne, M. A., Emerald Group Publishing, 319–385.

- Sala, L. (2015). "DSGE Models in the Frequency Domain." Journal of Applied Econometrics 30(2), 219–240.
- Sargent, T. J., and Surico, P. (2011). "Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals." *American Economic Review* 101(1), 109–128.
- Watson, M. W. (1993). "Measures of Fit for Calibrated Models." *Journal of Political Economy* 101(6), 1011–1041.

# 3 Inference with weakly dependent data

## Theory

\* Hayashi: chapters 2, 6.5–6.6.

Brockwell and Davis: chapters 6–7.

Davidson: chapters 13–14, 24.

Hamilton: chapter 7.

# GMM, moment matching

\* Hayashi: chapters 7.1–7.4.

- Hansen, L. P., Heaton, J., and Yaron, A. (1996). "Finite-Sample Properties of Some Alternative GMM Estimators." *Journal of Business & Economic Statistics* 14(3), 262–280.
- Hansen, L. P., and Heckman, J. J. (1996). "The empirical foundations of calibration." Journal of Economic Perspectives 10(1), 87–104.
- Hansen, L. P., and Singleton, K. (1991). "Computing Semiparametric Efficiency Bounds for Linear Time Series Models." In Nonparametric and semiparametric methods in econometrics and statistics: Proceedings of the Fifth International Symposium in Economic Theory and Econometrics, edited by Barnett, W. A., Powell, J., and Tauchen, G. E., Cambridge University Press, chapter 15, 388–411.
- Kydland, F., and Prescott, E. (1996). "The computational experiment: an econometric tool." *Journal of Economic Perspectives* 10(1), 69–85.

- Nakamura, E., and Steinsson, J. (2018). "Identification in Macroeconomics." *Journal of Economic Perspectives* 32(3), 59–86.
- Newey, W. K., and McFadden, D. L. (1994). "Large sample estimation and hypothesis testing." In *Handbook of Econometrics, Volume IV*, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 36, 2111–2245.

# **Bootstrap**

- \* Kilian and Lütkepohl: chapters 12.1–12.5.
  - Brüggemann, R., Jentsch, C., and Trenkler, C. (2016). "Inference in VARs with conditional heteroskedasticity of unknown form." *Journal of Econometrics* 191(1), 69–85.
  - Gonçalves, S., and Kilian, L. (2004). "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form." *Journal of Econometrics* 123(1), 89–120.
  - Horowitz, J. L. (2001). "The Bootstrap." In *Handbook of Econometrics, Volume 5*, edited by Heckman, J. J., and Leamer, E., Elsevier, chapter 52, 3159–3228.
  - Kilian, L. (1998). "Small-sample Confidence Intervals for Impulse Response Functions." Review of Economics and Statistics 80(2), 218–230.

#### Weak identification

- \* Andrews, I., Stock, J. H., and Sun, L. (2019). "Weak Instruments in Instrumental Variables Regression: Theory and Practice." *Annual Review of Economics* 11(1), 727–753.
  - Kleibergen, F., and Mavroeidis, S. (2009). "Weak Instrument Robust Tests in GMM and the New Keynesian Phillips Curve." *Journal of Business & Economic Statistics* 27(3), 293–339. With comments and rejoinder.

### Applications

- Christiano, L., Eichenbaum, M., and Evans, C. (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113(1), 1–45.
- Hansen, L. P., and Singleton, K. J. (1982). "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectation Models." *Econometrica* 50(5), 1269–1286.

- Mankiw, N. G., Reis, R., and Wolfers, J. (2004). "Disagreement about Inflation Expectations." In *NBER Macroeconomics Annual 2003, Volume 18*, edited by Gertler, M., and Rogoff, K., National Bureau of Economic Research, 209–248.
- Mavroeidis, S., Plagborg-Møller, M., and Stock, J. H. (2014). "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve." *Journal of Economic Literature* 52(1), 124–188.

# 4 Causal identification in macroeconomics

# Exclusion restrictions, instruments/proxies

- \* Stock, J. H., and Watson, M. W. (2018). "Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments." *Economic Journal* 128(610), 917–948.
  - Kilian and Lütkepohl: chapters 4, 7–12, 15.
  - Barnichon, R., and Brownlees, C. (2019). "Impulse Response Estimation By Smooth Local Projections." *Review of Economics and Statistics* 101(3), 522–530.
  - Blanchard, O., and Quah, D. (1989). "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review* 79(4), 655–673.
  - Jordà, O. (2005). "Estimation and Inference of Impulse Responses by Local Projections." American Economic Review 95(1), 161–182.
  - Li, D., Plagborg-Møller, M., and Wolf, C. K. (2023). "Local Projections vs. VARs: Lessons From Thousands of DGPs." Manuscript, Princeton University.
  - McKay, A., and Wolf, C. K. (2023). "What Can Time Series Regressions Tell Us About Policy Counterfactuals?" *Econometrica* 91(5), 1695–1725
  - Montiel Olea, J. L., Stock, J. H., and Watson, M. W. (2021). "Inference in SVARs Identified with External Instruments." *Journal of Econometrics* 225(1), 74–87.
  - Plagborg-Møller, M. (2019). "Bayesian Inference on Structural Impulse Response Functions." *Quantitative Economics* 10(1), 145–184.

- Plagborg-Møller, M., and Wolf, C. K. (2021). "Local Projections and VARs Estimate the Same Impulse Responses." *Econometrica* 89(2), 955–980.
- Rubio-Ramírez, J. F., Waggoner, D. F., and Zha, T. (2010). "Structural vector autoregressions: Theory of identification and algorithms for inference." *Review of Economic Studies* 77(2), 665–696.
- Sims, C. A. (1980). "Macroeconomics and Reality." Econometrica 48(1), 1–48.
- Uhlig, H. (2005). "What are the effects of monetary policy on output? Results from an agnostic identification procedure." *Journal of Monetary Economics* 52(2), 381–419.

# Invertibility, recoverability

- Chahrour, R., and Jurado, K. (2021). "Recoverability and Expectations-Driven Fluctuations." *Review of Economic Studies* 89(1), 214–239.
- Fernández-Villaverde, J., Rubio-Ramírez, J. F., Sargent T. J., and Watson, M. W. (2007). "ABCs (and Ds) of Understanding VARs." *American Economic Review* 97(3), 1021–1026.
- Forni, M., Gambetti, L., and Sala, L. (2019). "Structural VARs and noninvertible macroe-conomic models." *Journal of Applied Econometrics* 34(2), 221–246.
- Lippi, M., and Reichlin, L. (1994). "VAR analysis, nonfundamental representations, Blaschke matrices." *Journal of Econometrics* 63(1), 307–325.
- Plagborg-Møller, M., and Wolf, C. K. (2022). "Instrumental Variable Identification of Dynamic Variance Decompositions." *Journal of Political Economy* 130(8), 2164–2202.

#### Sign/magnitude restrictions

- \* Baumeister, C., and Hamilton, J. D. (2015). "Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information." *Econometrica* 83(5), 1963–1999.
  - Gafarov, B., Meier, M., and Montiel Olea, J. L. (2018). "Delta-Method Inference for a Class of Set-Identified SVARs." *Journal of Econometrics* 203(2), 316–327.
  - Giacomini, R., and Kitagawa, T. (2021). "Robust Bayesian Inference for Set-Identified Models." *Econometrica* 89(4), 1519–1556.

Wolf, C. K. (2020). "SVAR (Mis)Identification and the Real Effects of Monetary Policy." American Economic Journal: Macroeconomics 12(4), 1–32.

# Identification through non-Gaussianity/heteroskedasticity

- \* Montiel Olea, J. L., Plagborg-Møller, M., and Qian, E. (2022). "SVAR Identification From Higher Moments: Has the Simultaneous Causality Problem Been Solved?" *AEA Papers and Proceedings* 112, 481–485.
  - Kilian and Lütkepohl: chapter 14.
  - Gouriéroux, C., Monfort, A., and Renne, J.-P. (2017). "Statistical inference for independent component analysis: Application to structural VAR models." *Journal of Econometrics* 196(1), 111–126.
  - Gouriéroux, C., Monfort, A., and Renne, J.-P. (2020). "Identification and Estimation in Non-Fundamental Structural VARMA Models." *Review of Economic Studies* 87(4), 1915–1953.
  - Rigobon, R. (2003). "Identification Through Heteroskedasticity." Review of Economics and Statistics 85(4), 777–792.

#### Nonlinear models

- Angrist, J. D., Jordà, O., Kuersteiner, G. M. (2018). "Semiparametric Estimates of Monetary Policy Effects: String Theory Revisited." *Journal of Business & Economic Statistics* 36(3), 371–387.
- Angrist, J. D., and Pischke, J.-S. (2009). *Mostly Harmless Econometrics*. Princeton University Press. Chapter 3.
- Gallant, A. R., Rossi, P. E., and Tauchen, G. (1993). "Nonlinear dynamic structures." *Econometrica* 61(4), 871–907.
- Gonçalves, S., Herrera, A. M., Kilian, L., and Pesavento, E. (2023). "State-dependent local projections." Manuscript, McGill University.
- Koop, G., Pesaran, M. H., and Potter, S. M. (1996). "Impulse response analysis in nonlinear multivariate models." *Journal of Econometrics* 74(1), 119–147.

Rambachan, A., and Shephard, N. (2021). "When do Common Time Series Estimands have Nonparametric Causal Meaning?" Manuscript, Harvard University.

## **Applications**

- \* Ramey, V. A. (2016). "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics, Volume 2A*, edited by Taylor, J. B., and Uhlig, H., Elsevier, chapter 2, 71–162.
  - Forni, M., Gambetti, L., Lippi, M., and Sala, L. (2017). "Noisy News in Business Cycles." American Economic Journal: Macroeconomics 9(4), 122–152.
  - Gertler, M., and Karadi, P. (2015). "Monetary Policy Surprises, Credit Costs, and Economic Activity." *American Economic Journal: Macroeconomics* 7(1), 44–76.
  - Mertens, K., and Ravn, M. O. (2010). "Measuring the Impact of Fiscal Policy in the Face of Anticipation: A Structural VAR Approach." *Economic Journal* 120(544), 393–413.
  - Mertens, K., and Ravn, M. O. (2013). "The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States." *American Economic Review* 103(4), 1212–1247.
  - Ramey, V. A., and Zubairy, S. (2018). "Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data." *Journal of Political Economy* 126(2), 850–901.

# 5 State space models

## Linear state space models, DSGE estimation

- \* Hamilton: chapter 13.
- \* Herbst and Schorfheide: chapters 1–3.
- \* Griffoli, T. M. (2013). "Dynare User Guide." Sections 3, 5.
  - Blanchard, O. J., and Kahn, C. M. (1980). "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48(5), 1305–1311.
  - Durbin, J., and Koopman, S. J. (2012). *Time Series Analysis by State Space Methods*. 2nd edition, Oxford University Press.

Sims, C. A. (2002). "Solving Linear Rational Expectations Models." Computational Economics 20(1–2), 1–20.

#### Markov Chain Monte Carlo

\* Chib, S. (2001). "Markov Chain Monte Carlo Methods: Computation and Inference." In *Handbook of Econometrics, Volume 5*, edited by Heckman, J. J., and Leamer, E., Elsevier, chapter 5, 3564–3634.

Herbst and Schorfheide: chapter 4.

## Regime switching

Hamilton: chapter 22.4.

# Applications

Sims, C. A., and Zha, T. (2006). "Were There Regime Switches in U.S. Monetary Policy?" American Economic Review 96(1), 54–81.

Smets, F., and Wouters, R. (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97(3), 586–606.

# 6 Dynamic factor models

#### Estimation and inference

\* Stock, J. H., and Watson, M. W. (2016). "Dynamic Factor Models, Factor-Augmented Vector Autoregressions, and Structural Vector Autoregressions in Macroeconomics." In *Handbook of Macroeconomics, Volume 2A*, edited by Taylor, J. B., and Uhlig, H., Elsevier, chapter 8, 415–525. Sections 1–3, 5–6.

Bai, J. (2003). "Inferential Theory for Factor Models of Large Dimensions." *Econometrica* 71(1), 135–171.

Bai, J., and Ng, S. (2008), "Large Dimensional Factor Analysis." Foundations and Trends in Econometrics 3(2), 89–163.

- Doz, C., Giannone, D., and Reichlin, L. (2012). "A Quasi-Maximum Likelihood Approach for Large, Approximate Dynamic Factor Models." *Review of Economics and Statistics*, 94(4), 1014–1024.
- Forni, M., Giannone, D., Lippi, M. and Reichlin, L. (2009). "Opening the Black Box: Structural Factor Models with Large Cross Sections." *Econometric Theory* 25(5), 1319–1347.
- Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). "The Generalized Dynamic-Factor Model: Identification and Estimation." *Review of Economics and Statistics* 82(4), 540–554.
- Stock, J. H., and Watson, M. W. (2002). "Forecasting Using Principal Components From a Large Number of Predictors." *Journal of the American Statistical Association* 97(460), 1167–1179.

#### Determining the number of factors

- Bai, J., and Ng, S. (2002). "Determining the Number of Factors in Approximate Factor Models." *Econometrica* 70(1), 191–221.
- Onatski, A. (2009). "Testing Hypotheses About The Number of Factors in Large Factor Models." *Econometrica* 77(5), 1447–1479.

### **Applications**

- Almuzara, M., Baker, K., O'Keeffe, H., and Sbordone, A. (2023). "The New York Fed Staff Nowcast 2.0." New York Fed Staff Technical Paper.
- Bernanke, B. S., Boivin, J., and Eliasz, P. (2005). "Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach." *Quarterly Journal of Economics* 120(1), 387–422.
- Bok, B., Caratelli, D., Giannone, D., Sbordone, A. M., and Tambalotti, A. (2017). "Macroeconomic Nowcasting and Forecasting with Big Data." *Annual Review of Economics* 10, 615–643.

# 7 Estimating heterogeneous agent models

- \* Winberry, T. (2018). "A method for solving and estimating heterogeneous agent macro models." *Quantitative Economics* 9(3), 1123–1151.
  - Ahn, S., Kaplan, G., Moll, B., Winberry, T., and Wolf, C. K. (2017). "When Inequality Matters for Macro and Macro Matters for Inequality." *NBER Macroeconomics Annual* 2017, edited by Eichenbaum, M., and Parker, J. A., chapter 1, 1–75.
  - Auclert, A., Bardóczy, B., Rognline, M., and Straub, L. (2021). "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models." *Econometrica* 89(5), 2375–2408.
  - Chang, M., Chen, X., and Schorfheide, F. (2023). "Heterogeneity and Aggregate Fluctuations." Manuscript, University of Pennsylvania.
  - Cocci, M. D., and Plagborg-Møller, M. (2023). "Standard Errors for Calibrated Parameters." Manuscript, Princeton University.
  - Hahn, J., Kuersteiner, G., and Mazzocco, M. (2020). "Estimation with Aggregate Shocks." Review of Economic Studies 87(3), 1365–1398.
  - Liu, L., and Plagborg-Møller, M. (2023). "Full-Information Estimation of Heterogeneous Agent Models Using Macro and Micro Data." *Quantitative Economics* 14(1), 2023, 1–35.

### 8 Functional Central Limit Theorem

## Abstract theory

Davidson: chapters 26–30.

- Andrews, D. W. K. (1994). "Empirical process methods in econometrics." In *Handbook of Econometrics*, Volume IV, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 37, 2247–2294.
- Hall, P., and Heyde, C. C. (1980). *Martingale Limit Theory and Its Application*. Academic Press. Chapter 4.
- Phillips, P. C. B., and Solo, V. (1992). "Asymptotics for Linear Processes." *Annals of Statistics* 20(2), 971–1001.

### Structural breaks

- Andrews, D. W. K., (1993). "Tests for Parameter Instability and Structural Change with Unknown Change Point." *Econometrica* 61(4), 821–856.
- Andrews, D. W. K. and Ploberger, W. (1994). "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative." *Econometrica* 62(6), 1383–1414.
- Bai, J. (1997). "Estimation of a Change Point in Multiple Regression Models." Review of Economics and Statistics 79(4), 551–563.
- Bai, J. (1997). "Estimating multiple breaks one at a time," *Econometric Theory* 13(3), 315–352.
- Bai, J., and Perron, P. (1998). "Estimating and Testing Linear Models with Multiple Structural Changes." *Econometrica* 66(1), 47–78.
- Bai, J., and Perron, P. (2003). "Computation and Analysis of Multiple Structural Change Models." *Journal of Applied Econometrics* 18, 1–22.
- Elliott, G., and Müller, U. K. (2006). "Efficient Tests for General Persistent Time Variation in Regression Coefficients." Review of Economic Studies 73(4), 907–940.
- Elliott, G., Müller, U. K., and Watson, M. W. (2015). "Nearly Optimal Tests When a Nuisance Parameter Is Present Under the Null Hypothesis." *Econometrica* 83(2), 771–811.
- Müller, U. K. and Petalas, P.-E. (2010). "Efficient Estimation of the Parameter Path in Unstable Time Series Models." *Review of Economic Studies* 77(4), 1508–1539.
- Nyblom, J. (1989). "Testing for the Constancy of Parameters Over Time." *Journal of the American Statistical Association* 84(405), 223–230.
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## **Applications**

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