Online Supplement to
"Standard Errors for Calibrated Parameters"

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In Section 5.2 of the main paper, we estimate parameters in a one-asset HANK model. This appendix gives further details on that model and the estimation exercise we perform. The model we estimate is nearly identical to Auclert, Bardóczy, Rognlie & Straub (2021) as described in their Appendix B.2 and estimated in their Appendix F.2.[1] The purpose of this document is to restate the full model of Auclert et al. (2021) and define the small number of departures from that baseline that we adopt for our empirical application.

Note that, in order to stay as close to Auclert et al. (2021) as possible, the notation in this online supplement conflicts with some of the notation in Sections 2–4 of our main paper.

1 Model summary and empirical specification

This section summarizes the equilibrium conditions of the one-asset HANK model in Appendix B of Auclert et al. (2021) that we adopt for the application in Section 5.2 of our main paper. We explain in detail the minor changes that we make to the specification of the Taylor rule and exogenous shock processes relative to Auclert et al. (2021). Finally, we specify the parameters that we estimate. For ease of access, Section 2 below reviews the full micro foundations for the model, as also explained in Auclert et al. (2021).

Equilibrium conditions. The one-asset HANK model in Appendix B of Auclert et al. (2021) consists of a unit mass of heterogeneous households that spend, save, and borrow (up to a limit), a unit mass of monopolistically competitive intermediate goods firms, a

[1] That model is itself adapted from McKay, Nakamura & Steinsson (2016).

competitive final goods market, a monetary authority responsible for setting the nominal interest rate, and a fiscal authority responsible for taxation and government spending. The equilibrium conditions for model aggregates are given by

$$F_t(\boldsymbol{X},\boldsymbol{Z}) = \begin{pmatrix} Y_t - Z_t N_t \\ Y_t - \frac{\mu_t}{\mu_t - 1}\frac{1}{2\kappa}\log(1+\pi_t)^2 Y_t - w_t N_t - d_t \\ r_t B + G_t - \tau_t \\ r_t^* + \phi\pi_t + \phi_y(Y_t - Y_{ss}) - i_t \\ 1 + r_t - \frac{1+i_{t-1}}{1+\pi_t} \\ \kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu_t}\right) + \frac{1}{1+r_{t+1}}\frac{Y_{t+1}}{Y_t}\log(1+\pi_{t+1}) - \log(1+\pi_t) \\ \mathcal{A}_t(\{r_s, w_s, \tau_s, d_s\}) - B \\ \mathcal{N}_t(\{r_s, w_s, \tau_s, d_s\}) - N_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad (1)$$

where $\mathcal{A}_t(\cdot)$ and $\mathcal{N}_t(\cdot)$ denote the aggregate savings and labor supply across heterogeneous households given the interest rate, wage rate, taxes, and dividends. The equations represent (in order) production, dividends distributed, the government budget constraint, the Taylor rule, the Fisher equation, the Phillips curve, asset market clearing, and labor market clearing. The aggregate resource constraint is omitted by Walras' law. For further details on the model economy and equilibrium conditions, see either Appendix B of Auclert et al. (2021) or Section 2 of this document.

In the above equation, $\boldsymbol{X} = (\boldsymbol{X}_t)_{t\in\mathbb{Z}}$ denotes the sequence of endogenous aggregate variables, the period-t values of which are $\boldsymbol{X}_t = (Y_t, N_t, \pi_t, w_t, d_t, r_t, \tau_t, i_t)$, while $\boldsymbol{Z} = (\boldsymbol{Z}_t)_{t\in\mathbb{Z}}$ denotes the sequence of exogenous driving variables with $\boldsymbol{Z}_t = (Z_t, G_t, r_t^*, \mu_t)$. We specify the dynamics of the latter below.

Steady state. The steady state is defined as the (constant) value $\boldsymbol{X}_{ss}$ for $\boldsymbol{X}_t$ implied by the exogenous variables being equal to the constant values $\boldsymbol{Z}_{ss} = (Z, G, r, \mu)$ at all points in time. It is easy to see that this zero-inflation steady state does not depend on the parameters (κ, ϕ, ϕ_y). We follow Auclert et al. (2021) and assume that the values of the remaining model parameters are known and given by the values listed in Table 1.[2] We solve for the steady state at these parameter values, using exactly the same code as in Auclert et al. (2021).[3]

[2] Values are taken from their Table B.2.

[3] For brevity, we suppress details on the discretization of grids and dynamics for idiosyncratic states in the household problem.

Parameter		Value
$\beta = (1+r)^{-1}$	Discount factor	0.982
φ	Disutility of labor	0.786
σ	Inverse IES	2
ν	Inverse Frisch	2
μ	Steady-state markup	1.2
B	Bond supply	5.6
G	Steady state government spending	0
Z	Steady state TFP	1
$\underline{a}$	Borrowing limit	0
ρ_e	Autocorrelation of efficiency hours	0.966
σ_e	Efficiency hour shock std. dev.	$0.5\sqrt{1-\rho_e^2}$

Table 1: Values for non-estimated parameters. Note that some of these parameters only enter into the equilibrium conditions (1) through the aggregate household decision functions $\mathcal{A}_t(\cdot)$ and $\mathcal{N}_t(\cdot)$.

Exogenous disturbance processes. We now describe the processes we assume for the exogenous variables $\boldsymbol{Z}_t = (Z_t, G_t, r_t^*, \mu_t)$. Our specification generalizes that of Auclert et al. (2021), as discussed further below. Define log TFP growth $z_t = \log(Z_t/Z_{t-1})$, government spending in deviation from steady state as a fraction of steady state output $g_t = (G_t - G)/Y_{ss}$, and the markup and natural rate in deviation from steady state, $\tilde{\mu}_t = \mu_t - \mu$ and $\tilde{r}_t^* = r_t^* - r$. We assume the following exogenous AR(2) processes:

$$
\begin{aligned}
z_t &= \rho_{z1} z_{t-1} + \rho_{z2} z_{t-2} + \varepsilon_t^z, && \varepsilon_t^z \sim (0, \sigma_z^2), \\
g_t &= \rho_{g1} g_{t-1} + \rho_{g2} g_{t-2} + \varepsilon_t^g, && \varepsilon_t^g \sim (0, \sigma_g^2), \\
\tilde{r}_t^* &= \rho_{r1} \tilde{r}_{t-1}^* + \rho_{r2} \tilde{r}_{t-2}^* + \varepsilon_t^r, && \varepsilon_t^r \sim (0, \sigma_r^2), \\
\tilde{\mu}_t &= \rho_{\mu1} \tilde{\mu}_{t-1} + \rho_{\mu2} \tilde{\mu}_{t-2} + \varepsilon_t^\mu, && \varepsilon_t^\mu \sim (0, \sigma_\mu^2).
\end{aligned}
$$

All the shocks ε_t^j for $j \in \{z, g, r, \mu\}$ are i.i.d. and mutually independent.

Modifications relative to the Auclert et al. (2021) specification. Our empirical specification differs from that in Auclert et al. (2021, Table F.III) in the following ways.

First, the Auclert et al. (2021) model specification obtains if (i) we assume that there are no TFP shocks ($\sigma_z = 0$) and (ii) we restrict the exogenous variables to follow AR(1) processes ($\rho_{g2} = \rho_{r2} = \rho_{\mu2} = 0$). We drop all these assumptions and thereby strictly generalize their specification of the exogenous driving forces.

Second, while Auclert et al. (2021) estimate the Taylor rule parameter ϕ_y on output, we restrict this parameter to equal 0. This restriction is in line with the numerically small value of ϕ_y estimated by Auclert et al. (2021, Table F.III). The restriction is also imposed in the variant of the one-asset HANK model featured in the online Python toolbox developed by Auclert et al.[4]

In summary, the minor modifications we make relative to Auclert et al. (2021) strictly generalize their assumptions on the exogenous driving processes, but impose the restriction that the Taylor rule depends only on inflation and not output. These are the only differences in model specification between our application and that of Auclert et al. (2021, Table F.III). Recall that these changes do not affect the model's steady state.

Estimated parameters. The 7 parameters we estimate are: the Taylor rule coefficient on inflation ϕ, the slope of the Phillips curve κ, the parameters of the autoregressive process for TFP growth $(\rho_{z1}, \rho_{z2}, \sigma_z)$, and the autoregressive coefficients for the monetary shock (ρ_{r1}, ρ_{r2}). We do not explicitly estimate the parameters in the autoregressive processes for the government spending disturbance g_t and the markup disturbance $\tilde{\mu}_t$, as these parameters do not influence the particular impulse responses that we match (see the definition of the matched impulse responses in our main paper). Note, however, that our estimation procedure allows for any values of these non-estimated parameters, as explained in our main paper.[5] We similarly do not estimate the standard deviation of the monetary shock σ_r, since this parameter does not affect the *normalized* impulse responses with respect to monetary shocks that we match (see the definition of the matched impulse responses in our main paper). Again, we note that our estimation procedure allows for any value of σ_r, though this parameter is not directly estimated.

Auclert et al. (2021) estimate all parameters of the government spending, markup, and monetary disturbance processes, though they restrict these processes to be autoregressions of order 1 rather than order 2. The fact that they estimate these parameters, however, does not mean that they allow for more shocks in their model than we do. Our impulse response matching procedure is robust to the presence of other shocks, including the government spending and markup shocks explicitly defined above.

[4] https://github.com/shade-econ/sequence-jacobian

[5] In fact, we do not even need to restrict these processes to be autoregressions of order 2.

2 Detailed model specification

This section gives a full accounting of the microfoundations of the one-asset HANK model, the equilibrium conditions of which were summarized above. This section merely reviews the model of Appendix B.2 of Auclert et al. (2021), whose modeling assumptions we adopt without any changes (recall that the only changes we make to the empirical specification are to the exogenous disturbance processes, as defined earlier).

Households. Each heterogeneous household's problem is characterized by the following Bellman equation:

$$
\begin{aligned}
V_t(e_{it}, a_{it-1}) = \max_{c_{it}, n_{it}, a_{it}} &\left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t V_{t+1}(e_{it+1}, a_{it}) \right\} \\
c_{it} + a_{it} &= (1 + r_t) a_{it-1} + w_t e_{it} n_{it} - \tau_t \bar{\tau}(e_{it}) + d_t \bar{d}(e_{it}) \\
a_{it} &\geq \underline{a}
\end{aligned}
$$

Specifically, each household optimally chooses its real consumption c_{it}, labor supply n_{it}, and real asset holdings a_{it} subject to a budget and borrowing constraint $a_{it} \geq \underline{a}$. Income for consumption and savings is generated from three sources. First, households can supply labor n_{it}, for which they earn a wage rate w_t per efficiency hour e_{it}. Households' idiosyncratic efficiency state e_{it} evolves exogenously over time, and the probability of transitioning from state e to state e' is governed by a joint probability distribution denoted by $P(e, e')$. This transition kernel comes from a discretization of an AR(1) process for $\log e_{it} = \rho_e \log e_{i,t-1} + \epsilon_{i,t}$, $\epsilon_{i,t} \sim N(0, \sigma_e^2)$. Second, households have access to (real) principal and interest $(1 + r_t) a_{it-1}$ given their previous period asset holdings. Third, households receive dividend income $d_t \bar{d}(e_{it})$ from owning monopolistically competitive intermediate goods firms, to be discussed below. Subtracting from income, households must pay taxes $\tau_t \bar{\tau}(e_{it})$ in each period. Note that $\bar{d}(\,\cdot\,)$ and $\bar{\tau}(\,\cdot\,)$ are densities integrating to one; therefore, d_t and τ_t control the overall level.

Firms. First, aggregate output is produced by a single representative final goods firm in a competitive market that aggregates intermediate goods y_{jt} produced by a unit mass of monopolistically competitive intermediate goods firms according to

$$
Y_t = \left(\int_0^1 y_{jt}^{1/\mu_t} \, dj \right)^{\mu_t},
$$

where μ_t is an exogenous markup process. Let P_t denote the aggregate price level for final goods (more on this below). Let p_{jt} denote the price of the jth intermediate good. Taking the price of final goods P_t and demand Y_t as given,[6] the final good firm determines how to procure specific intermediate goods by solving the (nominal) profit maximization problem

$$\max_{y_{jt}} P_t Y_t - \int_0^1 p_{jt} y_{jt} \, dj \qquad \text{s.t.} \quad Y_t = \left(\int_0^1 y_{jt}^{1/\mu_t} \, dj \right)^{\mu_t}. \tag{2}$$

Next, each intermediate goods firm j produces y_{jt} according to production function

$$F(n_{jt}) = Z_t n_{jt},$$

where Z_t is aggregate TFP and n_{jt} is the efficiency units of labor hired by intermediate firm j.[7] Therefore, to produce y_{jt} units of intermediate output, the firm must hire $n_{jt} = y_{jt}/Z_t$ efficiency units of labor. Each intermediate firm can set their price in period t subject to a quadratic (real) adjustment cost

$$\psi_t(p_{jt}, p_{jt-1}) = \frac{\mu_t}{\mu_t - 1} \frac{1}{2\kappa} \log \left(\frac{p_{jt}}{p_{jt-1}} \right)^2 Y_t.$$

Under perfect foresight, taking as given demand schedules $y_{jt}(p_{jt})$ for intermediate good j, firm j chooses prices to maximize discounted (real) profits

$$\max_{\{p_{j,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} M_{t,t+s} \left\{ \frac{p_{j,t+s}}{P_{t+s}} y_{j,t+s}(p_{jt}) - w_{t+s} n_{j,t+s} \frac{y_{j,t+s}(p_{jt})}{Z_{t+s}} - \frac{\mu_{t+s}}{\mu_{t+s} - 1} \frac{1}{2\kappa} \log \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} \right)^2 Y_{t+s} \right\}, \tag{3}$$

where $M_{t,t+s}$ is the real discount factor, which comes from the household problem, since intermediate goods firms are owned by households.

Policy. The monetary authority sets the nominal interest rate i_t according to a Taylor rule

$$i_t = r_t^* + \phi \pi_t + \phi_y (Y_t - Y_{ss}), \tag{4}$$

[6] These will be pinned down later by market clearing and zero profits. At this point, the task is only to determine how Y_t is created from intermediate goods.

[7] This is a distinct definition from a household's n_{it} supply. Whereas firms' n_{jt} is a measure of efficiency units hired, n_{it} is raw hours, before scaling by efficiency. We handle this distinction in market clearing.

where r_t^* is the exogenous natural interest rate with steady state value $r = \beta^{-1} - 1$, $1 + \pi_t = P_t/P_{t-1}$ is inflation in the nominal price index, and Y_{ss} is steady state aggregate output. This rule, together with inflation, determines the real interest rate r_t via the Fisher equation

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}. \tag{5}$$

Explicitly, the real rate of interest paid out in period t on the asset purchased in period $t-1$ equals the promised nominal rate at time i_{t-1} less realized inflation π_t between $t-1$ and t.

Finally, the fiscal authority has exogenous spending G_t and pays interest at rate r_t on the outstanding stock of bonds with constant real face value B.

Equilibrium. Equilibrium is characterized by (1) a sequence of policy functions and value functions for the household problem $\{c_t(e, a_{-1}), n_t(e, a_{-1}), a_t(e, a_{-1}), V_t(e, a_{-1})\}_{t=0}^{\infty}$, (2) a sequence of distributions $\{\Gamma_t(e, a)\}_{t=0}^{\infty}$ over the idiosyncratic states with support $\mathcal{E} \times \mathcal{A}$, (3) and aggregate sequences $\{Y_t, N_t, \pi_t, w_t, d_t, r_t, \tau_t, i_t, \psi_t\}_{t=0}^{\infty}$ consistent with equilibrium conditions, given the exogenous processes. We now state these equilibrium conditions.

First, bond market clearing requires that aggregate savings by households equals the stock of outstanding bonds in each period,

$$B = \int_{\mathcal{E}} \int_{\mathcal{A}} a_t(e, a_{-1}) d\Gamma_t(e, a_{-1}). \tag{6}$$

The government must also balance its budget in each period,

$$r_t B + G_t = \int_{\mathcal{E}} \int_{\mathcal{A}} \tau_t \bar{\tau}(e)\, d\Gamma_t(e, a) = \tau_t, \tag{7}$$

where τ_t is aggregate taxes.

On the firm side, by standard arguments, there are zero profits in the competitive final goods market and there is a symmetric equilibrium in the intermediate goods market in which all monopolistically competitive intermediate goods firms set identical prices, hire an identical amount of labor, and produce an identical amount of output. For brevity, we focus only upon the aggregate implications of equilibrium in the production side of the model and suppress details at the level of individual firms. Therefore, let P_t denote the price level in period t, let N_t denote the aggregate efficiency units of labor hired across all firms, and let Y_t denote the corresponding amount of the final good produced. Equilibrium in the intermediate goods market implies the following Phillips curve

$$\log(1+\pi_t) = \kappa\left[\frac{w_t}{Z_t} - \frac{1}{\mu_t}\right] + \frac{1}{1+r_{t+1}}\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}. \tag{8}$$

Second, real aggregate output of the final good must equal aggregate production

$$Y_t = Z_t N_t. \tag{9}$$

Lastly on the firm side, aggregate real dividends remitted to households equals total real output less labor and price adjustment costs,

$$\int_{\mathcal{E}}\int_{\mathcal{A}} d_t\bar{d}(e)\, d\Gamma(e,a) = d_t = Y_t - w_t N_t - \frac{\mu_t}{\mu_t - 1}\frac{1}{2\kappa}\log(1+\pi_t)^2\, Y_t. \tag{10}$$

Next, labor market clearing requires labor demand from firms equals households' supply,

$$N_t := \int_{\mathcal{E}}\int_{\mathcal{A}} e\, n_t(e,a_{-1})\, d\Gamma_t(e,a_{-1}). \tag{11}$$

The aggregate resource constraint requires that private consumption equals total output less government spending and aggregate price adjustment costs,

$$\int_{\mathcal{E}}\int_{\mathcal{A}} c_t(e,a_{-1}) d\Gamma_t(e,a_{-1}) = Y_t - G_t - \frac{\mu_t}{\mu_t - 1}\frac{1}{2\kappa}\log(1+\pi_t)^2. \tag{12}$$

This last equation can be derived by aggregating the budget constraint of the individual heterogeneous households and using Equations (6), (7), (10), and (11) to simplify.

Finally, the distributions must be consistent with the policy functions of households,

$$\Gamma_{t+1}(e',a) = \int_{\mathcal{E}}\int_{\mathcal{A}} \mathbf{1}\{a_t(e,a_{-1}) = a\} P(e'|e) d\Gamma_t(e,a_{-1}).$$

Sequence form. Stacking the equilibrium conditions in Equations (4)–(11), we obtain the system of equations (1) described in the previous section. The model is solved using the linearization technique developed by Auclert et al. (2021).

References

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